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Gravitational effects induced by high-power lasers

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Abstract. The gravitational field produced by a high-power laser is calculated according to the linearized Einstein field equation in weak field approximation. It is found that when a probe pulse propagates in the above gravitational field, the gravitational Faraday effect and defocusing phenomenon of the probe pulse are induced by the gradient of the energy density of the high-power laser pulse. The rotating angle of the polarization plane of the probe pulse and the variation of its light intensity are derived. The results are discussed and estimated under the conditions of our present experiment facility.

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1 Introduction

It is well-known that the plane of polarization of a light ray undergoes a rotation as it propagates through a plasma in the presence of a magnetic field, which is called the electromagnetic Faraday effect. The gravitational Faraday effect is a gravitational analog of the usual Faraday effect due to the structure of the curved spacetime. This effect was first discussed by Skrotskii [1] and many authors have examined this gravitational effect under different gravitational backgrounds [2–4]. Under the curved spacetime background, the orbit of light rays will deflect because light rays parallel transport along the null geodesics in the curved spacetime [5] and the plane of polarization of light rotates. Carini et al. [6] discussed the rotation of the plane of polarization and its intensity variation under the Kerr spacetime background.

Relativistic gravitational effects mostly concern physical phenomena in large scale spacetime. With the rapid development of high-power laser technology in recent years, the intensity of a laser pulse comes up to the order of magnitude above $10^{20}\,\mathrm{W/cm^2}$, so that its energy density is strong enough to result in the spacetime nearby being curved, which provides a gravitational field equivalent. Thus, we can hope to detect gravitational effects predicted by general relativistic theory in the laboratory. The study of general relativistic effects in the gravitational field produced by high-power lasers goes back to 1979, when Scully [7] considered the deflection and phase shifting of a probe pulse in the above gravitational field. We also examined the energy shifting of hydrogen atoms [8] in the curved spacetime induced by high-power lasers.

In this paper, we will study some behaviors for a probe pulse propagating in the curved spacetime induced by high-power laser using the approach developed by Carini [6]. These behaviors include two effects, one is the gravitational Faraday effect in this gravitational background and another is the defocusing phenomenon of probe pulse induced by the gradient of the energy density of high-power laser pulse when it travels near the high-power laser with the same velocity.

Our research in this paper is as follows. In Sect. 2, the gravitational field produced by a high-power laser is calculated using the linearized Einstein field equation in the weak field approximation. In Sect. 3, we adopt Hanni's definition of Maxwell equations in the curved spacetime [9] and Carini's transform of Schrödinger-like equation [6] to derive the probe's wave function in this gravitational field under geometrical optics approximation. In Sect. 4, we assume the probe is a beam of linearly polarized light which can be described as the linear superposition of positive helicity state and negative helicity state. We calculate the phase shifts of two helicity states respectively. It shows that the phase shifts of these two helicity states are different, hence the plane of polarization of this linear polarized light superposed by these two helicity states will appear a rotating angle. This is the gravitational Faraday effect in this gravitational background. We also derive the probe's defocusing phenomenon when this probe pulse travels near the high-power laser with the same velocity. Finally, the result obtained is discussed and estimated numerically. Throughout this paper, we use the natural units in which $\hbar = c = 1$ and we take the signature $(1, -1, -1, -1)$ for the spacetime metric.

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2 The gravitational field produced by high-power laser

The gravitational field caused by the energy-momentum of high-power lasers belongs to weak field. The metric can be expressed as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \tag{1}$$

where $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ is Minkowski metric of a flat spacetime, and $h_{\mu\nu}$ denotes the perturbation describing a curved spacetime, which can be calculated by linearized Einstein field equation

$$\Box h_{\mu\nu} = 16\pi G\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right). \tag{2}$$

Here $\Box = -\partial_\mu\partial^\mu$ denotes D'Alembert operator, G is gravitational constant, $T = T^\alpha{}_\alpha$ and the energy-momentum tensor $T_{\mu\nu}$ is defined as

$$T_{\mu\nu} = \frac{1}{4\pi}\left(F_\mu{}^\lambda F_{\lambda\nu} + \frac{1}{4}\eta_{\mu\nu}F^{\sigma\rho}F_{\sigma\rho}\right). \tag{3}$$

The covariant electromagnetic field tensor $F_{\mu\nu}$ is given by

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}, \tag{4}$$

and $F^{\mu\nu} = \eta^{\mu\rho}\eta^{\nu\sigma}F_{\rho\sigma}$, $F_\mu{}^\lambda = \eta^{\lambda\sigma}F_{\mu\sigma}$.

We assume that the high-power laser pulse is propagating in $+z$ direction with velocity of v and consider that this pulse as propagating down such a TE wave guide in a transverse electric configuration. Thus, its electric field and magnetic field can be shown as

$$E_x = \varepsilon(\mathbf{r},t)\sin(\omega t - kz)\,, \tag{5}$$
$$B_y = \varepsilon(\mathbf{r},t)v\sin(\omega t - kz)\,, \tag{6}$$
$$B_z = \varepsilon(\mathbf{r},t)\left(1-v^2\right)^{\frac{1}{2}}\cos(\omega t - kz)\,, \tag{7}$$

where $\varepsilon(\mathbf{r},t)$ denotes the envelope of pulse given by

$$\varepsilon^2(\mathbf{r},t) = E_0^2 A[\theta(v(t+T)-z) - \theta(vt-z)]\delta(x)\delta(y)\,, \tag{8}$$

where E_0 is the amplitude of laser pulse, A denotes the cross-section of laser pulse and T represents the duration of laser pulse. The step function $\theta(z)$ is defined as

$$\theta(z) = \begin{cases} 1, & z > 0 \\ 0, & z < 0 \end{cases}. \tag{9}$$

Inserting (5)–(7) into (3), we obtain 9 nonzero components of energy-momentum tensors $T_{\mu\nu}$ as follows

$$T_{00} = \frac{1}{8\pi}\varepsilon^2(\mathbf{r},t)\left[\left(1-v^2\right) + 2v^2\sin^2(\omega t - kz)\right], \tag{10}$$
$$T_{20} = T_{02} = \frac{1}{8\pi}\varepsilon^2(\mathbf{r},t)\left(1-v^2\right)^{\frac{1}{2}}\sin 2(\omega t - kz)\,, \tag{11}$$
$$T_{30} = T_{03} = -\frac{1}{4\pi}\varepsilon^2(\mathbf{r},t)v\sin^2(\omega t - kz)\,, \tag{12}$$
$$T_{11} = \frac{1}{8\pi}\varepsilon^2(\mathbf{r},t)\left(1-v^2\right)\left[1 - 2\sin^2(\omega t - kz)\right], \tag{13}$$
$$T_{22} = \frac{1}{8\pi}\varepsilon^2(\mathbf{r},t)\left(1-v^2\right), \tag{14}$$
$$T_{32} = T_{23} = -\frac{1}{8\pi}\varepsilon^2(\mathbf{r},t)v\left(1-v^2\right)^{\frac{1}{2}}\sin 2(\omega t - kz)\,, \tag{15}$$
$$T_{33} = \frac{1}{8\pi}\varepsilon^2(\mathbf{r},t)\left[-\left(1-v^2\right) + 2\sin^2(\omega t - kz)\right]. \tag{16}$$

Because the trace of $T_{\mu\nu}$ is defined as $T^\alpha{}_\nu = \eta^{\alpha\mu}T_{\mu\nu}$, therefore using (10)–(16), we can prove that the trace of $T_{\mu\nu}$ vanishes, i.e. $T = T^\alpha{}_\alpha = 0$. Hence, linearized Einstein field (2) becomes

$$\Box h_{\mu\nu} = 16\pi G T_{\mu\nu}\,. \tag{17}$$

Here the phase factor $\omega t - kz$ can be written as $-k(z - v_{ph}t)$ in which v_{ph} is the phase velocity of high-power laser pulse. From the expression of $T_{\mu\nu}$ we note that the energy-momentum tensor can be expressed as $T_{\mu\nu}(x,y,z-v_{ph}t)$. If we assume that the medium is non-dispersion, the pulse's group velocity v and the phase velocity v_{ph} are identical. Thus, $T_{\mu\nu}(x,y,z-v_{ph}t)$ can be rewritten as $T_{\mu\nu}(x,y,z-vt)$. We introduce a co-moving coordinate $\xi = z - vt$ and then the D'Alembert operator can be written as

$$-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(1-v^2\right)\frac{\partial^2}{\partial \xi^2}\,. \tag{18}$$

Assuming

$$x = \bar{x}\,, \tag{19}$$
$$y = \bar{y}\,, \tag{20}$$
$$\xi = \left(1-v^2\right)^{\frac{1}{2}}\bar{\xi}\,, \tag{21}$$

we can recast (18) in the form of

$$\bar{\nabla}^2 = \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2} + \frac{\partial^2}{\partial \bar{\xi}^2}\,, \tag{22}$$

hence the simplified linearized Einstein field(17) becomes

$$\bar{\nabla}^2 h_{\mu\nu} = 16\pi G T_{\mu\nu}\,. \tag{23}$$

From the (23) we note that the simplified linearized Einstein field equation have the same form of Poisson equation. We solve the equation of $\bar{\nabla}^2 h_{00} = 16\pi G T_{00}$ firstly.

The solution is

$$
\begin{aligned}
h_{00} &= -\frac{1}{4\pi}(16\pi G\rho A)\left[\left(1-v^2\right)+2v^2\sin^2(kvt')\right] \\
&\quad\times\int_{-\infty}^{+\infty} d\overline{\xi}' \int_{-\infty}^{+\infty} d\overline{y}' \int_{-\infty}^{+\infty} d\overline{x}' \\
&\quad\times\frac{[\theta(vT-\left(1-v^2\right)^{\frac{1}{2}}\overline{\xi}')-\theta(\left(1-v^2\right)^{\frac{1}{2}}\overline{\xi}')]\delta(\overline{y}')\delta(\overline{x}')}{[(\overline{x}-\overline{x}')^2+(\overline{y}-\overline{y}')^2+(\overline{\xi}-\overline{\xi}')^2]^{\frac{1}{2}}} \\
&= (-4G\rho A)\left[\left(1-v^2\right)+2v^2\sin^2(kvt')\right] \\
&\quad\times\int_0^{(1-v^2)^{-\frac{1}{2}}vT}\frac{d\overline{\xi}'}{[\overline{x}'^2+\overline{y}'^2+(\overline{\xi}-\overline{\xi}')^2]^{\frac{1}{2}}} \\
&= \left[\left(1-v^2\right)+2v^2\sin^2(kvt')\right]h(\mathbf{r},t)\,,
\end{aligned} \tag{24}
$$

where

$$
h(\mathbf{r},t) = (-4G\rho A)\times \ln\left(\frac{[z-v(t+T)]+\{[z-v(t+T)]^2+(1-v^2)(x^2+y^2)\}^{\frac{1}{2}}}{(z-vt)+[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{1}{2}}}\right),
$$

and ρ is defined as $\rho = E_0^2/8\pi$, which is the radiation energy density of high-power laser. To obtain the maximum accumulated effect of the phenomenon we assume that the probe pulse propagates along $+z$ direction with the same group velocity v in the width of high-power laser pulse just like riding on it by the probe, i.e. the displacement difference between the probe pulse and the high-power laser pulse $z-vt$ can be considered as a constant. We also consider the term of trigonometric function $\sin^2\left(-k\left(z-vt\right)\right)$ as a constant term of $\sin^2(kvt')$ in which t' is a constant between 0 and the laser pulse duration T. The reason for this approximation is that the wavelength of high-power laser pulse can be compared with the displacement difference $z-vt$ so that the variation of $\sin^2(-k(z-vt))$ almost vanishes in a short time interval.

Analogically, we can derive the rest nonzero components of $h_{\mu\nu}$ as follows

$$
h_{02} = h_{20} = -\left(1-v^2\right)^{\frac{1}{2}}\sin(2kvt')h(\mathbf{r},t)\,, \tag{25}
$$

$$
h_{03} = h_{30} = -2v\sin^2(kvt')h(\mathbf{r},t)\,, \tag{26}
$$

$$
h_{11} = \left(1-v^2\right)\left[1-2\sin^2(kvt')\right]h(\mathbf{r},t)\,, \tag{27}
$$

$$
h_{22} = \left(1-v^2\right)h(\mathbf{r},t)\,, \tag{28}
$$

$$
h_{23} = h_{32} = v\left(1-v^2\right)^{\frac{1}{2}}\sin(2kvt')h(\mathbf{r},t)\,, \tag{29}
$$

$$
h_{33} = \left[-\left(1-v^2\right)+2\sin^2(kvt')\right]h(\mathbf{r},t)\,. \tag{30}
$$

For the ultrashort laser pulse, we have $vT/\left[(z-vt)^2+\left(1-v^2\right)\left(x^2+y^2\right)\right]^{\frac{1}{2}} \ll 1$, hence $h(\mathbf{r},t)$ can be simplified as

$$
h(\mathbf{r},t) = \frac{-4G\rho V}{[(z-vt)^2+(1-v^2)\left(x^2+y^2\right)]^{\frac{1}{2}}}\,, \tag{31}
$$

where $V = AvT$ is the volume of the laser pulse. Thus, ρV denotes the energy of high-power laser pulse. Because the energy of high-power laser pulse completely concentrates in its pulse width which is extremely short, the effective range of perturbation $h(\mathbf{r},t)$ is only confined in the width of high-power laser pulse and moves with the same step as the pulse. This also can be seen from the perturbation expression (31). In another view, the (31) resembles the Coulomb potential excited by a moving electron, but here is the spacetime curvature induced by the moving high-power laser pulse.

3 The Maxwell equations in curved space

3.1 General form of Maxwell equations in curved space

There have been several ways to describe Maxwell equations in curved space according to the different definitions of electromagnetic vectors. Here, we adopt Hanni's definitions [9], the spatial metric is defined as

$$
\gamma_{ij} = -g_{ij}\,, \tag{32}
$$

$$
\gamma^{ij} = -g^{ij}+\frac{g^{0i}g^{0j}}{g^{00}}\,, \tag{33}
$$

$$
g^i = \frac{g^{i0}}{g^{00}}\,, \tag{34}
$$

$$
g_i = g_{0i}\,. \tag{35}
$$

According to the definition of electromagnetic tensor $F_{\mu\nu}$, we have

$$
E_i = F_{i0}\,, \tag{36}
$$

$$
B^i = \frac{\epsilon^{ijk}F_{jk}}{2\sqrt{\gamma}}\,, \tag{37}
$$

$$
D^i = \frac{F^{0i}}{\sqrt{g^{00}}}\,, \tag{38}
$$

$$
H_i = \frac{\sqrt{\gamma}\epsilon_{ijk}F^{jk}}{2\sqrt{g^{00}}}\,, \tag{39}
$$

where γ is the determinant of the 3D metric γ_{ij}. Maxwell equations can be written in the noncovariant form

$$
\nabla\times\mathbf{E} = -\frac{1}{\sqrt{\gamma}}\frac{\partial\left(\sqrt{\gamma}\mathbf{B}\right)}{\partial t}\,, \tag{40}
$$

$$
\nabla\times\mathbf{H} = \frac{1}{\sqrt{\gamma}}\frac{\partial\left(\sqrt{\gamma}\mathbf{D}\right)}{\partial t}\,, \tag{41}
$$

$$
\nabla\cdot\mathbf{B} = 0\,, \tag{42}
$$

$$
\nabla\cdot\mathbf{D} = 0\,, \tag{43}
$$

with the constitutive equations

$$
\mathbf{D} = \left(g^{00}\right)^{\frac{1}{2}}(\mathbf{E}+\mathbf{g}\times\mathbf{B})\,, \tag{44}
$$

$$
\mathbf{B} = \left(g^{00}\right)^{\frac{1}{2}}(\mathbf{H}+\mathbf{D}\times\mathbf{g})\,. \tag{45}
$$

Carini et al. [6] define the photon's wave function as $|\mathbf{\Psi}\rangle = \mathbf{D}+i\mathbf{B}$ and they transform Maxwell (40)–(43) into the form of a Schrödinger-like equation by using this definition

$$i\frac{\partial}{\partial t}|\mathbf{\Psi}\rangle = \nabla \times \left(|\mathbf{\Psi}\rangle/\sqrt{g^{00}} + i\mathbf{g} \times |\mathbf{\Psi}\rangle\right) , \qquad (46)$$

together with the transverse condition

$$\nabla \cdot |\mathbf{\Psi}\rangle = 0 . \qquad (47)$$

By denoting $g = \left(g^{00}\gamma\right)^{-\frac{1}{2}}$ and making the replacement $|\mathbf{\Psi}\rangle \to |\mathbf{\Psi}\rangle/\sqrt{\gamma}$, (46) becomes

$$i\frac{\partial}{\partial t}|\mathbf{\Psi}\rangle = g(\mathbf{s}\cdot\hat{\mathbf{K}})|\mathbf{\Psi}\rangle^c + (\mathbf{g}\cdot\hat{\mathbf{K}})|\mathbf{\Psi}\rangle + \nabla g \times |\mathbf{\Psi}\rangle^c + i\left(|\mathbf{\Psi}\rangle\cdot\nabla\right)\mathbf{g} - i(\nabla\cdot\mathbf{g})|\mathbf{\Psi}\rangle , \qquad (48)$$

where $\hat{\mathbf{K}} = -i\nabla$ is the momentum operator, $\mathbf{s} = \{s_i\}$ is the photon's spin operator given by the adjoint representation of SO(3), i.e. $\left(s^i\right)^{jk} = -i\epsilon^{ijk}$, and $|\mathbf{\Psi}\rangle^c$ is the contravariant three-vector corresponding to $|\mathbf{\Psi}\rangle$, namely, $|\mathbf{\Psi}\rangle^c_i = \gamma_{ij}|\mathbf{\Psi}\rangle^j$. The transverse equation then takes the form $\hat{\mathbf{K}}\cdot|\mathbf{\Psi}\rangle = 0$.

3.2 The photon's wave function in curved space under geometrical optics approximation

Under the geometrical optics approximation, the wave function of photon $\mathbf{\Psi}(\mathbf{r},t)$ characterized by a typical wave number k and a frequency ω, which are large enough in comparison to the spatial and temporal rates of variation of the propagating medium, can be regarded as a plane wave locally. The space curvature induced by high-power laser is so weak that its radius of curvature R is a large number, therefore, we can denote the relation between R and k as follows

$$kR \sim \epsilon^{-1} \gg 1 ,$$

where ϵ is a small dimensionless parameter. Thus, we can expand the photon's wave function $\mathbf{\Psi}(\mathbf{r},t)$ in the powers of ϵ,

$$|\mathbf{\Psi}\rangle = \sum_{n=0} \epsilon^n |\mathbf{\Psi}_n\rangle \exp\left(\frac{i\Phi}{\epsilon}\right) , \qquad (49)$$

and then define the local wave vector and frequency under geometrical optics approximation as

$$\mathbf{k}(\mathbf{r},t) = \nabla\Phi(\mathbf{r},t) , \qquad (50)$$

$$\omega(\mathbf{r},t) = -\partial_t\Phi(\mathbf{r},t) . \qquad (51)$$

Inserting (49) into (48) and keeping the zeroth order only, we derive

$$H_0|\mathbf{\Psi}_0\rangle = \omega|\mathbf{\Psi}_0\rangle , \qquad (52)$$

where $H_0|\mathbf{\Psi}_0\rangle = g(\mathbf{k}\cdot\mathbf{s})|\mathbf{\Psi}_0\rangle + (\mathbf{g}\cdot\mathbf{k})|\mathbf{\Psi}_0\rangle$. (52) is the eigenstate equation for a photon in the gravitational field induced by high-power laser. If the space is flat, the parameters g and $\mathbf{g}$ can be calculated readily, i.e. $g = 1$, $\mathbf{g} = \mathbf{0}$, then, (52) reduces to the form of $(\mathbf{k}\cdot\mathbf{s})|\mathbf{\Psi}_0\rangle = \omega|\mathbf{\Psi}_0\rangle$. This equation which is familiar to us describes the eigenstate equation for a photon in the flat space and it has two eigenvalues, the positive helicity state with the energy eigenvalue $\omega = |\mathbf{k}|$ and negative helicity state with $\omega = -|\mathbf{k}|$. These two helicity states correspond to the right and left polarization states of the photon respectively. However, the positivity of energy allows us to have only the positive state. For the negative energy state we may redefine the wave function as $|\mathbf{\Psi}\rangle = \mathbf{D} - i\mathbf{B}$, therefore the wave function becomes $(\mathbf{k}\cdot\mathbf{s})|\mathbf{\Psi}_0\rangle = -\omega|\mathbf{\Psi}_0\rangle$.

Let the unit vectors $\{\mathbf{e}_i\}$, $i = 1,2,3$ form an orthogonal tetrad with $\mathbf{e}_3$ along the direction of the wave vector $\mathbf{k}$. Assuming that $|\mathbf{e}_\pm\rangle$ denotes the eigenstates above, then we have

$$(\mathbf{e}_3\cdot\mathbf{s})\,|\mathbf{e}_\pm\rangle = \pm|\mathbf{e}_\pm\rangle , \qquad (53)$$

here we suppose that the amplitude of photon's wave function is the unit magnitude and $|\mathbf{e}_\pm\rangle$ can be expressed in terms of the real vector basis $\{\mathbf{e}_i\}$

$$|\mathbf{e}_\pm\rangle = \mp\frac{1}{\sqrt{2}}\left(\mathbf{e}_1 \pm i\mathbf{e}_2\right) , \qquad (54)$$

hence, the wave function of the photon can be written as a linear superposition of the two helicity states $|\mathbf{e}_\pm\rangle$

$$|\mathbf{\Psi}_0\rangle = \xi|\mathbf{e}_+\rangle + \eta|\mathbf{e}_-\rangle , \qquad (55)$$

where ξ and η satisfy the normalized condition.

4 Faraday effect in the gravitational field produced by high-power laser pulse and probe's defocusing phenomenon

Under the geometrical optics approximation, the wave function only depends on its propagating path length. Let C be the integral curve of $\mathbf{k}$ and the parameter s denote the path length of this curve. Introducing a phase factor term $|\mathbf{\Psi}\rangle = \varphi(s)|\mathbf{\Psi}_0\rangle$ and inserting it into (48), we have

$$i\frac{\partial}{\partial t}\left(|\mathbf{\Psi}_0\rangle\varphi\right) = g(\mathbf{s}\cdot\hat{\mathbf{K}})\left(|\mathbf{\Psi}_0\rangle^c\varphi\right) + (\mathbf{g}\cdot\hat{\mathbf{K}})\left(|\mathbf{\Psi}_0\rangle\varphi\right) + H_s\left(|\mathbf{\Psi}_0\rangle\varphi\right) , \qquad (56)$$

where $H_s\left(|\mathbf{\Psi}_0\rangle\varphi\right) = \nabla g \times \left(|\mathbf{\Psi}_0\rangle^c\varphi\right) + i(|\mathbf{\Psi}_0\rangle\varphi\cdot\nabla)\mathbf{g} - i(\nabla\cdot\mathbf{g})\left(|\mathbf{\Psi}_0\rangle\varphi\right)$.

Because the wave function only depends on its propagating path length, the derivative of $\varphi(s)|\mathbf{\Psi}_0\rangle$ with respect to t is zero, i.e.

$$i\frac{\partial}{\partial t}\left(|\mathbf{\Psi}_0\rangle\varphi\right) = 0 . \qquad (57)$$

Inserting (57) into (56) and replacing it with $\hat{\mathbf{K}} = -i\nabla$, we have

$$g(-i\mathbf{s}\cdot\nabla)\left(|\mathbf{\Psi}_0\rangle^c\varphi\right) + (-i\mathbf{g}\cdot\nabla)\left(|\mathbf{\Psi}_0\rangle\varphi\right) = -H_s\left(|\mathbf{\Psi}_0\rangle\varphi\right) . \qquad (58)$$

Let $\mathbf{n_k}$ be the unit wave vector, $\nabla = \mathbf{n_k}\partial_s$ and $\mathbf{n_k} = \nabla s$, then (58) becomes

$$ig\mathbf{n_k}\cdot\mathbf{s}\frac{\partial}{\partial s}\left(|\mathbf{\Psi}_0\rangle^c\varphi\right)+i\mathbf{g}\cdot\mathbf{n_k}\frac{\partial}{\partial s}\left(|\mathbf{\Psi}_0\rangle\varphi\right)=H_s\left(|\mathbf{\Psi}_0\rangle\varphi\right).$$

Here we approximate $|\mathbf{\Psi}_0\rangle^c=|\mathbf{\Psi}_0\rangle$ for only considering the first order of $h(\mathbf{r},t)$ and multiply the wave number k on both sides of above equation then we have

$$iH_0\frac{\partial}{\partial s}\left(|\mathbf{\Psi}_0\rangle\varphi\right)=kH_s\left(|\mathbf{\Psi}_0\rangle\varphi\right), \tag{59}$$

and take the inner product of both sides of the equation with $\langle\mathbf{\Psi}_0|$, we have

$$\frac{d\varphi}{ds}=-\frac{\langle\mathbf{\Psi}_0|H_0|\frac{d}{ds}\mathbf{\Psi}_0\rangle}{\langle\mathbf{\Psi}_0|H_0|\mathbf{\Psi}_0\rangle}\varphi-ik\frac{\langle\mathbf{\Psi}_0|H_s|\mathbf{\Psi}_0\rangle}{\langle\mathbf{\Psi}_0|H_0|\mathbf{\Psi}_0\rangle}\varphi, \tag{60}$$

where $H_s|\mathbf{\Psi}_0\rangle=\nabla g\times|\mathbf{\Psi}_0\rangle^c+i(|\mathbf{\Psi}_0\rangle\cdot\nabla)\mathbf{g}-i(\nabla\cdot\mathbf{g})|\mathbf{\Psi}_0\rangle$. Let the phase factor φ be exponential form $\varphi=\exp(i\gamma_p)$. Insert it into (60) and integrate, we obtain

$$\gamma_p=\gamma_{p_1}+\gamma_{p_2}, \tag{61}$$

where

$$\gamma_{p_1}=i\int_0^s ds\frac{\langle\mathbf{\Psi}_0|H_0|\frac{d}{ds}\mathbf{\Psi}_0\rangle}{\langle\mathbf{\Psi}_0|H_0|\mathbf{\Psi}_0\rangle}, \tag{62}$$

and

$$\gamma_{p_2}=-\int_0^s ds\frac{\langle\mathbf{\Psi}_0|H_s|\mathbf{\Psi}_0\rangle}{\langle\mathbf{\Psi}_0|H_0|\mathbf{\Psi}_0\rangle}k. \tag{63}$$

From (62) we find that γ_{p_1} denotes the Berry phase of the photon. In this paper, we chiefly discuss some behaviors induced by γ_{p_2}. We derive the result is that the term γ_{p_2} will lead to two phenomena one is the rotation of probe's plane of polarization in curved space produced by high-power laser pulse and the other is the decrease of intensity of the probe pulse via some computations. The thinking of these computations is that we suppose the probe pulse is a beam of linearly polarized light which is the superposition of the positive and the negative helicity states equally at first. Then we calculate their phase shift traveling in this gravitational background respectively and note that they are different. Finally after resuperposing these two helicity states which have different phase shift we obtain the results that the angle of probe's plane of polarization and its intensity will vary. The detailed calculations are as follows.

For calculating the integration in (63), we have to derive the quantities relevant to the matrix element $\langle\mathbf{\Psi}_0|H_s|\mathbf{\Psi}_0\rangle$ at first. In the present gravitational field, these relevant quantities are

$$\gamma=\det|\gamma_{ij}|=1-\left[\left(1-v^2\right)+2v^2\sin^2(kvt')\right]h(\mathbf{r},t), \tag{64}$$

$$g^{00}=1-\left[\left(1-v^2\right)+2v^2\sin^2(kvt')\right]h(\mathbf{r},t), \tag{65}$$

$$g=\left(g^{00}\gamma\right)^{-\frac{1}{2}}=1+\left[\left(1-v^2\right)+2v^2\sin^2(kvt')\right]h(\mathbf{r},t), \tag{66}$$

$$\mathbf{g}=\{g_{0i}\}=-\left(1-v^2\right)^{\frac{1}{2}}\sin(2kvt')h(\mathbf{r},t)\mathbf{e}_2 - 2v\sin^2(kvt')h(\mathbf{r},t)\mathbf{e}_3, \tag{67}$$

because of $h(\mathbf{r},t)\ll 1$, we take the first order of $h(\mathbf{r},t)$ in all of the above computations.

For the positive helicity state of the photon, we adopt $|\mathbf{\Psi}_0\rangle=|\mathbf{e}_+\rangle$ and the probe light propagates along with $+z$ direction. Insert the above definition and the relevant quantities (64)–(67) into the matrix element $\langle\mathbf{\Psi}_0|H_s|\mathbf{\Psi}_0\rangle$ and calculate all parts,

$$\begin{aligned}\langle\mathbf{\Psi}_0|\nabla g\times|\mathbf{\Psi}_0\rangle^c&=-i\frac{\partial}{\partial z}(\nabla g)\\&=-i\left[\left(1-v^2\right)+2v^2\sin^2(kvt')\right]\\&\quad\times\frac{(4G\rho V)(z-vt)}{[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{3}{2}}},\end{aligned} \tag{68}$$

$$\begin{aligned}i\langle\mathbf{\Psi}_0|(|\mathbf{\Psi}_0\rangle\cdot\nabla)\mathbf{g}&=-\frac{1}{2}\left(1-v^2\right)^{\frac{3}{2}}\sin(2kvt')\\&\quad\times\frac{(4G\rho V)x}{[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{3}{2}}}\\&\quad-i\left(1-v^2\right)^{\frac{3}{2}}\sin(2kvt')\\&\quad\times\frac{(4G\rho V)y}{[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{3}{2}}},\end{aligned} \tag{69}$$

$$\begin{aligned}-i\langle\mathbf{\Psi}_0|(\nabla\cdot\mathbf{g})|\mathbf{\Psi}_0\rangle&=i\left(1-v^2\right)^{\frac{3}{2}}\sin(2kvt')\\&\quad\times\frac{(4G\rho V)y}{[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{3}{2}}}\\&\quad+i2v\sin^2(kvt')\\&\quad\times\frac{(4G\rho V)(z-vt)}{[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{3}{2}}}.\end{aligned} \tag{70}$$

Insert (68), (69) and (70) into (63) and let $\gamma_{p_2}^+$ denote the γ_{p_2} effects induced by the positive helicity states of the photon. Because the probe pulse propagates along $+z$ direction with the group velocity v, the differential of the path length $s=vt$ can be expressed as $ds=vdt$. According to the Sect. 2, the term $z-vt$ can be seen as a constant independent of time. Hence, after integrating, we obtain

$$\gamma_{p_2}^+=\gamma_s^+-i\gamma_a^+, \tag{71}$$

where

$$\begin{aligned}\gamma_s^+&=\frac{1}{2}v\left(1-v^2\right)^{\frac{3}{2}}\sin(2kvt')\\&\quad\times\frac{(4G\rho V)x}{[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{3}{2}}}t,\end{aligned} \tag{72}$$

and

$$\begin{aligned}\gamma_a^+&=-v\left[\left(1-v^2\right)-2v(1-v)\sin^2(kvt')\right]\\&\quad\times\frac{(4G\rho V)(z-vt)}{[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{3}{2}}}t.\end{aligned} \tag{73}$$

For the case of negative helicity state of the photon, we adopt $|\mathbf{\Psi}_0\rangle=|\mathbf{e}_-\rangle$ and let $\gamma_{p_2}^-$ denote the γ_{p_2} effects induced by the negative helicity states of the photon analogically. We calculate it with the same method and obtain that

$$\gamma_{p_2}^- = \gamma_s^- - i\gamma_a^- \,, \tag{74}$$

where

$$\gamma_s^- = -\frac{1}{2}v\left(1-v^2\right)^{\frac{3}{2}}\sin(2kvt') \times \frac{(4G\rho V)x}{[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{3}{2}}}t\,, \tag{75}$$

and

$$\gamma_a^- = -v\left[\left(1-v^2\right)-2v(1-v)\sin^2(kvt')\right] \times \frac{(4G\rho V)(z-vt)}{[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{3}{2}}}t\,. \tag{76}$$

From (71)–(76), we note that $\gamma_{p_2}^+$ and $\gamma_{p_2}^-$ are different and they have two parts which are the real term and the imaginary term. Comparing them, we can find that the relations between their real terms and imaginary terms are $\gamma_s^+ = -\gamma_s^-$ and $\gamma_a^+ = \gamma_a^-$. It is evident that the real term shows the rotation angle of probe's plane of polarization. As to the imaginary term, it shows that the variation of intensity of probe pulse. For the linearly polarized light which is the superposition of the two helicity states equally, its rotation angle of probe's plane of polarization can be expressed as

$$\begin{aligned}\phi(t) &= \frac{1}{2}(\gamma_s^+ - \gamma_s^-)\\ &= \frac{1}{2}v\left(1-v^2\right)^{\frac{3}{2}}\sin(2kvt')\\ &\quad\times \frac{(4G\rho V)x}{[(z-vt)^2+(1-v^2)(x^2+y^2)]^{\frac{3}{2}}}t\,.\end{aligned} \tag{77}$$

It is the gravitational Faraday effect in the curved space induced by high-power laser pulse. The variation of probe's unit intensity is $I(t) = e^{2\gamma_a^+}I_0$, where $I_0 = \langle\mathbf{\Psi}_0|\mathbf{\Psi}_0\rangle$ is the unit input intensity of probe pulse. Because of $\gamma_a^+ \ll 1$, we can expand the term $e^{2\gamma_a^+}$ in terms of γ_a^+ and keep its first order only, hence $I(t)$ can be simplified as the linearized form

$$I(t) \simeq (1+2\gamma_a^+)I_0\,. \tag{78}$$

From the expression of γ_a^+, we can see that $\gamma_a^+ < 0$ i.e. $e^{\gamma_a^+} < 1$, it shows that the unit probe's intensity $I(t)$ is decreasing gradually along with its propagating direction. This is a defocusing phenomenon induced by the gradient of the energy density of high-power laser pulse.

5 Numerical estimate and discussion

From the above results, we find that the probe pulse can produce two effects, one is the gravitational Faraday effect of probe pulse and another is the probe's defocusing phenomenon induced by the gradient of the energy density of high-power laser pulse. To analyze our results further, we need to estimate them numerically and give a suggestion under the current experiment condition.

Because the probe pulse propagates in the same step as the high-power laser, we can manage to control the experiment to get the maximum effect. We are able to adjust $\sin(2kvt') = 1$ and try to emit a probe pulse at the same time after the high-power laser pulse emitted. Thus, $z-vt$ can be considered as the width of high-power laser pulse, x^2+y^2 can be seen as the effective cross-section of the laser pulse. If $x \sim y \sim r$, where r is the radius of the effective cross-section and $vT = z-vt$ denotes the width of the laser pulse, (77) and (78) can be written as

$$\phi(t) = \frac{1}{2}v\left(1-v^2\right)^{\frac{3}{2}}\frac{(4G\rho V)r}{[(vT)^2+(1-v^2)r^2]^{\frac{3}{2}}}t\,, \tag{79}$$

and

$$\varDelta I(t) = -I_0\left\{2\left[v(1-v)\frac{(4G\rho V)vT}{[(vT)^2+(1-v^2)r^2]^{\frac{3}{2}}}\right]\right\}t\,, \tag{80}$$

where $\varDelta I(t) = I(t) - I_0$ is the variation of $I(t)$. As what have mentioned in Sect. 2, ρV is the energy of high-power laser pulse. Setting $\rho V \sim 10^6$J, $r \sim 10^{-4}$m and $v = 0.9c$, the order of the rotating angle ϕ is approximately estimated as

$$\phi \sim (10^{-24}s^{-1})t\,, \tag{81}$$

and the variation of intensity is

$$\varDelta I \sim -I_0(10^{-23}s^{-1})t\,. \tag{82}$$

The above results indicate that the pulses must propagate a very long distance in order to get the measurable effects in the laboratory. It is difficult to travel such long distance in the present experiment condition. But if the pulses can be restricted in a ring wave guide, the gravitational effects can be amplified by prolonging the propagating time t of the pulses. With the development of laser technique and diagnose technology, we have the reason to believe that the realization of these two effects in the laboratory will be workable.

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